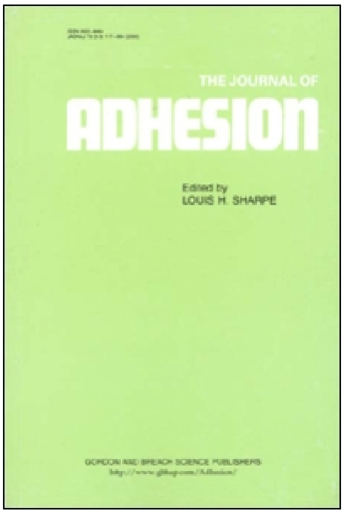

The Journal of Adhesion
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713453635

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To cite this Article Nagaraja, Y. R. and Alwar, R. S.(1979) 'Nonlinear Stress Analysis of an Adhesive Tubular Lap Joint', The Journal of Adhesion, 10: 2, 97 — 106
To link to this Article: DOI: 10.1080/00218467908544617
URL: http://dx.doi.org/10.1080/00218467908544617

J. Adhesion, 1979, Vol. 10, pp. 97–106
0021-8464/79/1002-0097 $04.50/0

Printed in Scotland

Nonlinear Stress Analysis of an Adhesive Tubular Lap Joint

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(*Received February 16, 1978; in final form January 2, 1979*)

The analysis of adhesive bonded joints have been based, generally, on the assumption that the adhesive behaves as a linearly elastic material. Many adhesives, however, exhibit nonlinear stress-strain behaviour, particularly near failure. This article reports an attempt on the analysis of an adhesive tubular lap joint with the adhesive obeying a nonlinear stress-strain law. The Finite Element method has been employed for the solution.

1. INTRODUCTION

The analysis of adhesive joints has been based, generally, on the linear theory of elasticity employing Hooke's law, since it is the simplest constitutive relationship for the behaviour of a material. However, the use of this law is justified only in certain cases like that of a material which obeys Hooke's law up to rupture. In practice, many adhesives exhibit nonlinear stress–strain behaviour particularly near the failure load. Hence, the analysis of an adhesive joint that has to account for the behaviour of the joint up to failure, has to take into consideration the nonlinear stress–strain behaviour of the adhesive. In addition, if the nonlinearity exists over a large portion of the stress–strain curve, then in order to have a true estimate of the stress state, even under working loads, a nonlinear analysis becomes necessary.

The most common means of characterization of an engineering material is through its stress–strain curve, which is generally obtained by a uniaxial tension or compression test on a standard specimen made out of the material. It should be noted that a stress–strain curve obtained from a uniaxial test relates only a single component of stress to a single component of strain. In order to represent stress–strain relations in biaxial and triaxial states of stress, a general triaxial nonlinear stress–strain law has to be used. The unknown parameters of such a law must be determined from the data provided by experimentally determined stress–strain curve.

In order to represent the nonlinear behaviour of the adhesive used in this investigation, a nonlinear biaxial stress–strain law suggested by Kauderer[1] has been employed. Kauderer himself has solved a large number of static nonlinear problems by using this type of stress–strain law. Subsequently, Evans and Pister,[2] and Byre Gowda and Topper[3] have used similar relationships for solving certain plane stress nonlinear problems. The constants appearing in the Kauderer law have been obtained from the data of a uniaxial tension test. It is assumed that the stress–strain relationship is independent of time and also that the state of stress is dependent only on the state of strain and not on the history of stress in the material.

With an adhesive whose nonlinear stress–strain relationship is represented by Kauderer's law, a tubular lap joint, with similar adherends and subjected to axial tension, has been analysed employing the finite element method.

2. NOTATIONS

T_0, T' = Hydrostatic and deviatoric stress tensors

D_0, D' = Dilatational and deviatoric strain tensors

σ_0, ε_0 = Mean normal stress and mean normal strain

K, G = Bulk modulus and shear modulus

$\varepsilon_r, \varepsilon_\theta, \varepsilon_z$ = Normal components of strain

$\sigma_r, \sigma_\theta, \sigma_z$ = Normal components of stress

$\varepsilon_{r\theta}, \varepsilon_{\theta z}, \varepsilon_{zr}$ = Shear strain components

$\sigma_{r\theta}, \sigma_{\theta z}, \sigma_{zr}$ = Shear stress components.

3. THE NONLINEAR STRESS–STRAIN LAW

The nonlinear biaxial stress–strain law proposed by Kauderer is expressed in terms of the dilatational and deviatoric stress and strain tensors as follows.

$$T_0 = 3K\chi(\varepsilon_0)D_0 \tag{1}$$

$$T' = 2G\gamma(\psi_0^2)D' \tag{2}$$

where

$$T_0 = \begin{pmatrix} \sigma_0 & 0 & 0 \\ 0 & \sigma_0 & 0 \\ 0 & 0 & \sigma_0 \end{pmatrix} \qquad D_0 = \begin{pmatrix} \varepsilon_0 & 0 & 0 \\ 0 & \varepsilon_0 & 0 \\ 0 & 0 & \varepsilon_0 \end{pmatrix}$$

$$T' = \begin{pmatrix} \sigma_r - \sigma_0 & \sigma_{r\theta} & \sigma_{rz} \\ \sigma_{r\theta} & \sigma_\theta - \sigma_0 & \sigma_{\theta z} \\ \sigma_{rz} & \sigma_{\theta z} & \sigma_z - \sigma_0 \end{pmatrix}$$

$$D' = \begin{pmatrix} \varepsilon_r - \varepsilon_0 & \frac{1}{2}\varepsilon_{r\theta} & \frac{1}{2}\varepsilon_{rz} \\ \frac{1}{2}\varepsilon_{r\theta} & \varepsilon_\theta - \varepsilon_0 & \frac{1}{2}\varepsilon_{\theta z} \\ \frac{1}{2}\varepsilon_{rz} & \frac{1}{2}\varepsilon_{\theta z} & \varepsilon_z - \varepsilon_0 \end{pmatrix}$$

$$\sigma_0 = \frac{\sigma_r + \sigma_\theta + \sigma_z}{3} \qquad \varepsilon_0 = \frac{\varepsilon_r + \varepsilon_\theta + \varepsilon_z}{3}$$

$$\psi_0^2 = \tfrac{4}{3}\left(\tfrac{2}{3}(\varepsilon_r^2 + \varepsilon_\theta^2 + \varepsilon_z^2 - \varepsilon_r\varepsilon_\theta - \varepsilon_r\varepsilon_z - \varepsilon_z\varepsilon_\theta) + \tfrac{1}{2}(\varepsilon_{r\theta}^2 + \varepsilon_{\theta z}^2 + \varepsilon_{rz}^2)\right)$$

and K and G are the bulk and shear modulii of the material respectively.

$\chi(\varepsilon_0)$ and $\gamma(\psi_0^2)$ are functions of ε_0 and $\psi_0^2\psi^2$ bound by the following conditions:

$$\operatorname*{Limit}_{\varepsilon_0 \to 0} \chi(\varepsilon_0) = 1, \qquad \operatorname*{Limit}_{\psi_0 \to 0} \gamma(\psi_0^2) = 1 \tag{3}$$

The above conditions make the Kauderer's law identical with Hooke's law for very small deformations. Conditions in Eq. (3) are satisfied if the functions $\chi(\varepsilon_0)$ and $\gamma(\psi_0^2)$ are assumed in the following form:

$$\chi(\varepsilon_0)\dagger = 1 + \chi_1\varepsilon_0 + \chi_2\varepsilon_0^2 + \chi_3\varepsilon_0^3 + \ldots \tag{4}$$

$$\gamma(\psi_0^2) = 1 + \gamma_2\psi_0^2 + \gamma_4\psi_0^4 + \gamma_6\psi_0^6 + \ldots \tag{5}$$

Using Eqs. (1) and (2) the relationship between the total stress tensor and the total strain tensor can be expressed as

$$T = T_0 + T' = 3K\chi(\varepsilon_0)D_0 + 2G\gamma(\psi_0^2)D' \tag{6}$$

For the case of axisymmetry the above relations reduce to

$$\begin{aligned} \sigma_r &= 3K\chi(\varepsilon_0)\varepsilon_0 + 2G\gamma(\psi_0^2)(\varepsilon_r - \varepsilon_0) \\ \sigma_\theta &= 3K\chi(\varepsilon_0)\varepsilon_0 + 2G\gamma(\psi_0^2)(\varepsilon_\theta - \varepsilon_0) \\ \sigma_z &= 3K\chi(\varepsilon_0)\varepsilon_0 + 2G\gamma(\psi_0^2)(\varepsilon_z - \varepsilon_0) \\ \sigma_{rz} &= G\gamma(\psi_0^2)\varepsilon_{rz} \end{aligned} \tag{7}$$

where

$$\psi_0^2 = \tfrac{4}{3}\left(\tfrac{2}{3}(\varepsilon_r^2 + \varepsilon_\theta^2 + \varepsilon_z^2 - \varepsilon_r\varepsilon_\theta - \varepsilon_\theta\varepsilon_z - \varepsilon_z\varepsilon_r) + \tfrac{1}{2}\varepsilon_{zr}^2\right).$$

† In the present analysis it is assumed that $\chi(\varepsilon_0) = 1$, i.e., the nonlinearity is assumed to be present only in the relation between the deviatoric stress and strain tensors.

The strains can be expressed in terms of the stresses as

$$\varepsilon_r = \frac{1}{3K}k(S_0)\sigma_0 + \frac{1}{2G}g(t_0^2)(\sigma_r - \sigma_0)$$

$$\varepsilon_\theta = \frac{1}{3K}k(S_0)\sigma_0 + \frac{1}{2G}g(t_0^2)(\sigma_\theta - \sigma_0)$$

$$\varepsilon_z = \frac{1}{3K}k(S_0)\sigma_0 + \frac{1}{2G}g(t_0^2)(\sigma_z - \sigma_0)$$

$$\varepsilon_{zr} = \frac{1}{G}g(t_0^2)\sigma_{zr} \tag{8}$$

where

$$S_0 = \sigma_0/3K$$
$$k(S_0) = 1 + k_1 S_0 + k_2 S_0^2 + k_3 S_0^3 + \dots$$
$$g(t_0^2) = 1 + g_2 t_0^2 + g_4 t_0^4 + g_6 t_0^6 + \dots$$
$$t_0^2 = \tau_0^2/G^2$$
$$\tau_0^2 = \tfrac{2}{3}\left(\tfrac{1}{3}(\sigma_r^2 + \sigma_\theta^2 + \sigma_z^2 - \sigma_r\sigma_\theta - \sigma_\theta\sigma_z - \sigma_z\sigma_r) + \sigma_{zr}^2\right) \tag{9}$$

Equations (7) and (8) form the basic nonlinear stress–strain relations for the case of axisymmetry as suggested by Kauderer.

For the solution of the present problem the functions $g(t_0^2)$ and $k(S_0)$ are assumed to be:

$$g(t_0^2) = 1 + g_2 t_0^2, \tag{10}$$

$$k(S_0) = 1 \tag{11}$$

where g_2 is given by[1]

$$g_2 = \left(1 + \frac{G}{3K}\right)\left(\frac{3G}{\sqrt{2}}\right)^2 a_3 \tag{12}$$

with a_3 determined from a uniaxial stress–strain curve obtained experimentally and using the equation[1]

$$\varepsilon_z = \frac{1}{E}(1 + a_3\sigma_z^2)\sigma_z \tag{13}$$

Knowing $g(t_0^2)$ $g(t^2)$ and $k(S_0)$ the three dimensional nonlinear stress–strain relations can be written using Eqs. (7) and (8).

4. EXPERIMENTAL STRESS–STRAIN CURVE

The adhesive employed in this investigation is V.P. 1527. The stress–strain curve for the material was obtained by testing a specimen in the laboratory under uniaxial tension. The specimen cut from a thin sheet was made to conform to the ASTM Standards[4] for testing of plastics. Figure 1 shows the stress–strain curve. The Young's modulus for the adhesive has been taken to be equal to the slope of the tangent to the initial portion of the stress–strain curve.

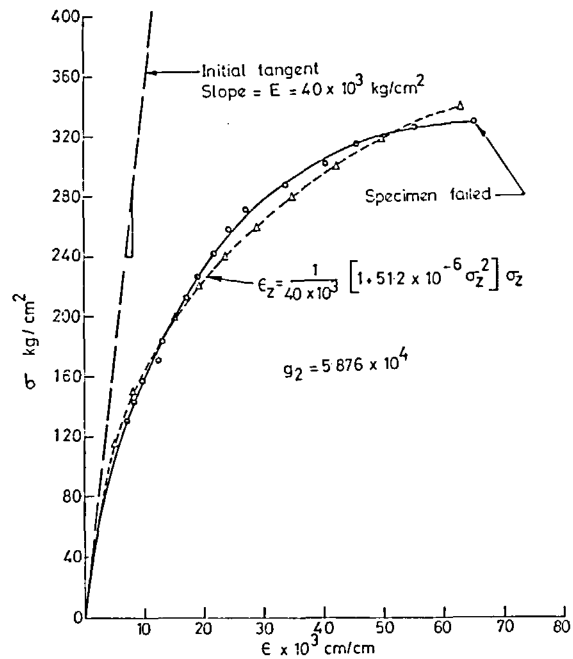

FIGURE 1 Stress–Strain curve for VP-1527 in Tension.

A one-point collocation has been performed with the above stress–strain curve in order to determine the constant a_3 and hence determine the value of g_2 given in Eqs. (13) and (12) respectively. With this value of g_2 the nonlinear stress–strain relationships given by Eq. (8) are completely determined. The collocated curve is also shown in Figure 1.

5. FINITE ELEMENT ANALYSIS

The finite element method originally developed for the analysis of linear systems has been successfully employed to analyse nonlinear systems with the aid of incremental and iterative techniques.

In the incremental load approach, the load is applied in small increments assuming that the material stiffness is constant during the application of each increment, but changing each element stiffness at the end of each load step in accordance with the current state of strain developed in the elements. The solution thus progresses through the use of a step-by-step procedure and is piecewise linear.

In the iterative approach the analysis begins with the determination of a linear solution at the maximum loading and used as a first approximation to determine the nonlinear values. The iterative procedure is continued until convergence occurs or the procedure fails to converge after a prescribed number of iterations. This procedure is satisfactory for slightly nonlinear systems. But for highly nonlinear systems, the solutions may begin to oscillate and convergence could be very slow.

The third method of solution is a combination of the incremental and the iterative procedures. Here, the analysis of the system has two operations, the first is analysing the system with the first increment of load, and the second operation is analysing the system at this load iteratively until convergence of results occurs. This procedure is continued until the maximum value of the load is reached. Thus, the nonlinear analysis is approximated as a sequence of linear steps. Since convergence is required at each load step, this procedure provides good results for highly nonlinear structures.

In the present analysis, the solution has been assumed to have converged when the difference in the values of the stresses, displacements, etc., between the current and the previous iterations is less than about 5%.

In the finite element method the stresses are determined from the relation

$$\sigma = [D]\varepsilon \tag{14}$$

The nonlinear stiffness matrix $[D]$ for analysing the tubular lap joint under axisymmetric conditions is given below.

$$\begin{Bmatrix} \sigma_r \\ \sigma_\theta \\ \sigma_z \\ \sigma_{zr} \end{Bmatrix} = \begin{bmatrix} D1 & D2 & D2 & 0 \\ D2 & D1 & D2 & 0 \\ D2 & D2 & D1 & 0 \\ 0 & 0 & 0 & D3 \end{bmatrix} \begin{Bmatrix} \varepsilon_r \\ \varepsilon_\theta \\ \varepsilon_z \\ \varepsilon_{zr} \end{Bmatrix} \tag{15}$$

where

$$\begin{aligned} D1 &= K+\tfrac{4}{3}G-\tfrac{32}{27}Gg_2\beta^2-\tfrac{8}{9}Gg_2\varepsilon_{zr}^2 \\ D2 &= K-\tfrac{2}{3}G+\tfrac{16}{27}Gg_2\beta^2+\tfrac{4}{9}Gg_2\varepsilon_{zr}^2 \\ D3 &= G-\tfrac{8}{9}Gg_2\beta^2-\tfrac{2}{3}Gg_2\varepsilon_{zr}^2 \end{aligned} \tag{16}$$

and

$$\beta^2 = \varepsilon_r^2+\varepsilon_\theta^2+\varepsilon_z^2-\varepsilon_r\varepsilon_\theta-\varepsilon_\theta\varepsilon_z-\varepsilon_z\varepsilon_r$$

It can be seen that the $[D]$ matrix in Eq. (14) is a function of the state of strain in the element and also a function of the initial elastic constants E and μ and of g_2.

6. RESULTS AND DISCUSSION†

Figure 2 shows the tubular lap joint analysed. The two adherends are similar and are assumed to be of steel, with their Young's modulii equal to 2.1×10^6 kg/cm^2 and their Poisson's ratio equal to 0.3. The adhesive, V.P. 1527, is represented by its collocated nonlinear stress–strain curve shown in Figure 1.

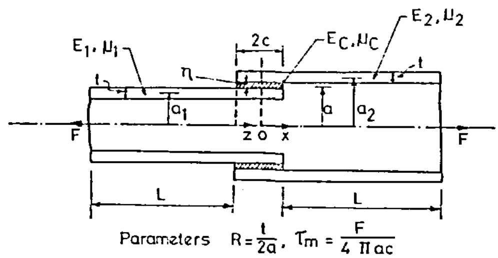

FIGURE 2 Tubular Lap Joint.

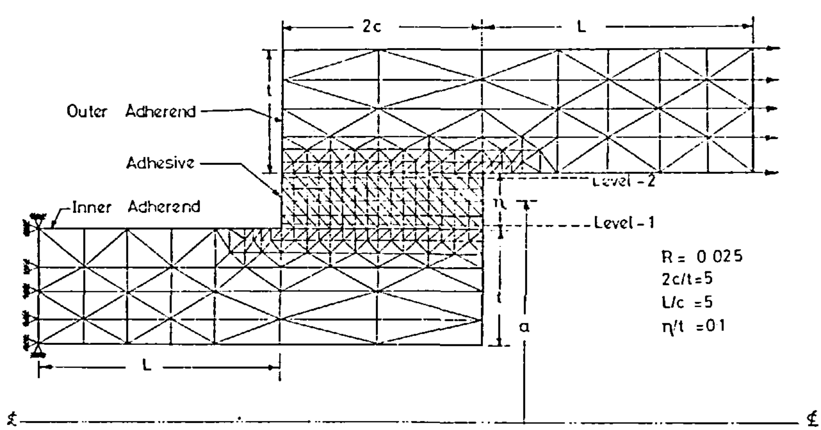

FIGURE 3 Finite Element Configuration (not to scale).

In order to analyse the joint by the Finite Element method the joint has been descretized as shown in Figure 3. In this figure the various parameters and the boundary conditions are also indicated.

Initially, the nonlinear stresses in the adhesive were obtained using the direct iterational approach. It was found that up to a load of 40,000 kg on

† All the numerical results of this investigation were obtained from the IBM System/370 Model-155 Computer at the I.I.T., Madras.

the joint the nonlinear stresses could be obtained by direct iteration procedure using the linear value as the initial guess. The results converged within 20 iterations and it was observed that the difference between two successive results was not more than 2 %. For loads higher than 40,000 kg an incremental procedure was employed.

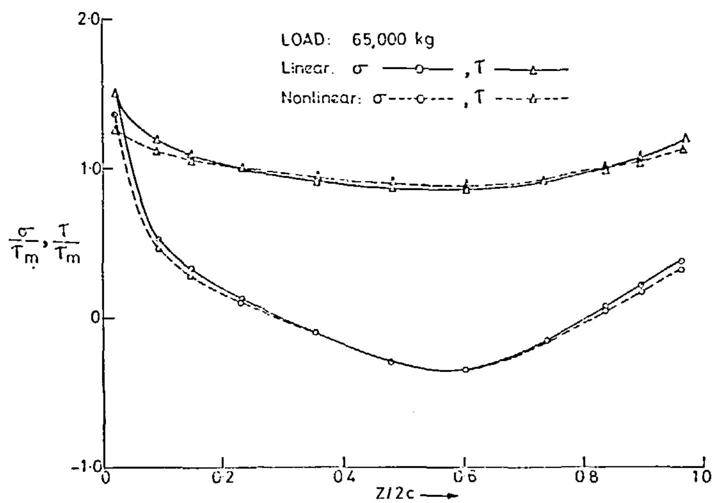

FIGURE 4 Variation of adhesive stresses along the overlap at level-1 in a tubular lap joint.

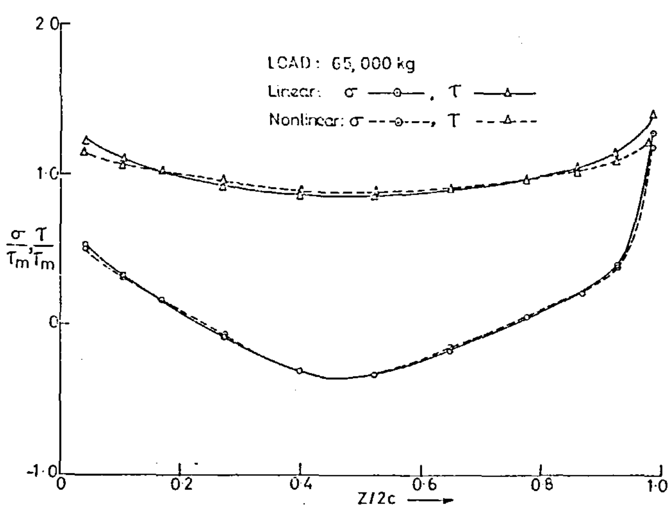

FIGURE 5 Variation of adhesive stresses along the overlap at level-2 in a tubular lap joint.

Starting with the nonlinear values at 40,000 kg, increments of 1000 kg and 500 kg were used to obtain the nonlinear values for loads up to 65,000 kg. Figure 4 illustrates the distributions of the adhesive normal and shear stresses, at level 1 (see Figure 3), for the maximum load of 65,000 kg. The stresses have been nondimensionalized by dividing them by the average shear stress τ_m, the value of which is indicated in Figure 2.

It can be observed from the above figure that there is considerable difference between linear and nonlinear stresses at the joint edges while the difference is not significant over the central region of the overlap.

Figure 5 shows the stress distributions at level 2 for the same load. From Figures 4 and 5 it is seen that the variation of stresses across the adhesive thickness is confined to the edges of the joint.

TABLE I

Stresses in the edge element at level 1 (kg/cm²)

Load (kg)	Linear		Nonlinear		Difference (%)	
	σ	τ	σ	τ	σ	τ
10,000	5.952	5.953	5.945	5.936	0.12	0.28
20,000	11.904	11.906	11.842	11.784	0.52	1.02
30,000	17.857	17.859	17.634	17.424	1.24	2.43
40,000	23.832	23.835	23.340	22.882	2.06	4.00
50,000	29.761	29.765	28.661	27.443	4.03	7.80
60,000	35.713	35.718	33.254	30.895	6.88	13.50
65,000	38.690	38.695	35.634	32.720	7.89	15.44

Fracture Stress = 332.2 kg/cm²

Table I gives the percentage difference between the linear and the nonlinear analyses regarding the normal and shear stresses near the edge of the overlap. It can be seen that even for low stress levels of the order of 12% of the fracture stress, there is significant difference between the linear and nonlinear values, around 8% for the normal stress and 15% for shear stress. This indicates that the design of an adhesive joint based on a conventional linear elastic analysis would definitely be on the safer side.

7. CONCLUSIONS

The finite element analysis of a tubular lap joint with the adhesive obeying a nonlinear biaxial stress–strain law indicates that there can be considerable difference between the linear and the nonlinear stresses even at loads of the order of 12% of the fracture load. To this extent the linear stress analysis is conservative in the sense that the calculated stresses will be higher than the true stresses. The difference between the linear and the nonlinear calculations could be augmented at higher load levels and therefore resulting in a greater factor of safety with regard to the linear analysis in the design of adhesive bonded joints.

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